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η' Mass and Chiral Symmetry Breaking at Large N_c and N_f

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Abstract

We propose a method for implementing the large-N_c, large-N_f limit of QCD at the effective Lagrangian level. Depending on the value of the ratio N_f/N_c, different patterns of chiral symmetry breaking can arise, leading in particular to different behaviors of the η'-mass in the combined large-N limit.

1. Large-N_c considerations successfully explain many non-perturbative aspects of confining gauge theories [1,2]. However, there are at least two exceptions -both related to a strong OZI rule violation- in which the $1/N_c$ expansion apparently fails: (i) In the scalar channel the spectrum is not dominated by a nonet of ideally mixed states and chiral symmetry breaking exhibits an important dependence on the number N_f of light quark flavors [3]; (ii) At large N_c, the η'-field becomes massless due to its relation to the U(1) anomaly while Nature realizes it like a heavy state. In this note we reconsider these problems in the limit in which both N_f and N_c tend to infinity with fixed ratio [4]. Since, at least at lowest orders of perturbation theory, the (rescaled) QCD β-function (with $g^2 N_c \equiv$ const.) only depends on the ratio N_f/N_c, we might expect that the hadronic spectrum resembles the physical one, in particular with chiral symmetry breakdown and $\Lambda_H \sim 1$ GeV. On the other hand, several hints (*e.g.* from the study of the conformal window, in QCD and its supersymmetric version), suggest a non trivial phase structure of the theory as a function of N_f and N_c. One can in principle distinguish three different phases, characterized by different symmetries of the vacuum, depending on the ratio N_f/N_c: (a) for low N_f/N_c only the $\mathrm{SU}_V(N_f)$ remains unbroken; (b) for higher N_f/N_c the vacuum is invariant under a larger group, $\mathrm{SU}_V(N_f) \times \mathrm{Z}_{\mathrm{chiral}}(N_f)$, where $\mathrm{Z}_{\mathrm{chiral}}(N_f)$ is the center of the chiral symmetry group $\mathrm{SU}_L(N_f)\times\mathrm{SU}_R(N_f)$ [5,6]; (c) for high N_f/N_c no spontaneous symmetry breaking takes place (and hence no confinement) and the symmetry of the vacuum is the whole $\mathrm{SU}_L(N_f)\times\mathrm{SU}_R(N_f)$. Notice that case ($b$) corresponds to the maximal possible symmetry of the vacuum in a confining vector-like theory. The existence of this phase is an assumption related to the issue of the non-perturbative renormalization of the bare Weingarten's inequalities comparing axial-axial and pseudoscalar-pseudoscalar two point functions [6,7]. We model the combined large-N_f, large-N_c limit by adding to the usual light flavors $q = (u, d, s)$, a set of N auxiliary flavors $Q = (Q_1, \ldots, Q_N)$ of common mass $M \gg m_q$, but still $M \ll \Lambda_H$. This mass M should be considered sufficiently small so that a power series expansion makes sense, but simultaneously much larger than any of the light quark masses m_q, thus the auxiliary fields can be integrated out at sufficiently low-energy. We then formally deal with $N_f = N + 3 \equiv n \to \infty$

flavors, but only the three lightest ones are physical. The rôle of these auxiliary flavors should be analogous to the one of the strange quark, when one considers the SU(2)×SU(2) chiral dynamics of u and d quarks.

2. Let N_f/N_c be subcritical, so that we are in the $\mathrm{Z}_{\text{chiral}}(n)$-asymmetric phase. The n^2-1 (pseudo) Goldstone bosons (GB) can be collected in a matrix $\hat{U}(x) \in \mathrm{SU}(n)$, (hereafter $n\times n$ matrices will be denoted by a hat) and their low-energy dynamics can be described by the effective Lagrangian

$$\mathcal{L}_{\text{sub}} = \frac{F^2}{4}\Big\{\langle D_\mu \hat{U} D^\mu \hat{U}^\dagger\rangle + 2B_0\langle \hat{U}^\dagger\hat{\chi} + \hat{\chi}^\dagger\hat{U}\rangle\Big\}\,. \tag{1}$$

where $\hat{\chi}$ is the scalar-pseudoscalar source,

$$\mathcal{L}_{\hat{\chi}}^{\text{QCD}} = -\bar{\Psi}_L\hat{\chi}\Psi_R - \bar{\Psi}_R\hat{\chi}^\dagger\Psi_L, \quad \Psi = \begin{pmatrix} q \\ Q \end{pmatrix}, \tag{2}$$

and $\langle\ldots\rangle$ denotes flavor trace. The $n\times n$ source matrix will be chosen as

$$\hat{\chi} = \begin{pmatrix} \chi & \mathbf{0} \\ \mathbf{0} & M\mathrm{e}^{i\theta/N}\mathbf{1}_{N\times N} \end{pmatrix}, \tag{3}$$

where χ is the 3×3 light quark source (mass term), θ is the vacuum angle and M is real and positive. Notice that there are no sources attached to the N auxiliary flavors and the corresponding GB degrees of freedom are frozen: in the tree approximation the $n\times n$ GB field matrix becomes

$$\hat{U} = \begin{pmatrix} U\mathrm{e}^{i\varphi/3} & \mathbf{0} \\ \mathbf{0} & \mathrm{e}^{-i\varphi/N}\mathbf{1}_{N\times N} \end{pmatrix} \in \mathrm{SU(n)}\,, \tag{4}$$

where $U\in\mathrm{SU}(3)$ collects the eight physical GB fields. The remaining U(1) field φ will be interpreted as the η'-field: for $\chi = m\mathbf{1}_{3\times 3}$ (no mixing),

$$\eta' = F\sqrt{\frac{n}{6N}}\varphi \tag{5}$$

and the corresponding mass, for $m=0$ is

$$M_{\eta'}^2 = \frac{6B_0M}{n} \to 0\,, \tag{6}$$

which vanishes in the (combined) large-N limit. Hence, *in this phase*, η'-mass behaves as in the standard $N_c \to \infty$, N_f-fixed limit.

3. For higher N_f/N_c we expect the $\mathrm{Z_{chiral}}(n)$-symmetric phase to occur: the vacuum is symmetric under

$$\Psi_{L,R} \to \mathrm{e}^{2\pi i\frac{k_{L,R}}{n}}\Psi_{L,R}, \quad k_{L,R} = 1, \ldots, n-1\,, \tag{7}$$

in addition to the usual $\mathrm{SU}_V(n)$. Notice that this $\mathrm{Z_{chiral}}(n)$ is also a subgroup of the (anomalous) $\mathrm{U}_L(1)\times\mathrm{U}_R(1)$. This additional symmetry of the vacuum finds its natural interpretation within the effective theory described by the Lagrangian $\mathcal{L}(\hat{U}, \hat{\chi}, \theta)$. The GB field $\hat{U}(x) \in \mathrm{SU}(n)$ is usually understood as simply connected to $\mathbf{1}$: $\hat{U}(x) = \mathbf{1} + i\varphi_a(x)T^a + \ldots$ and, in the corresponding effective theory, the integration measure $\mathcal{D}\hat{U}$ is treated accordingly. However $\mathrm{SU}(n)$ is *not* simply connected. We are free to choose an integration measure treating all sectors of $\mathrm{SU}(n)$ alike:

$$\int\mathcal{D}\hat{U}\mathrm{e}^{i\int dx\mathcal{L}(\hat{U},\hat{\chi},\theta)} \to \int\mathcal{D}\hat{U}\sum_{k=0}^{n-1}\mathrm{e}^{i\int dx\mathcal{L}(\hat{U}\mathrm{e}^{-2\pi ik/n},\hat{\chi},\theta)}\,, \tag{8}$$

where $\hat{U}$ and $\mathcal{D}\hat{U}$ again concern the connected vicinity of $\mathbf{1}$. This freedom derives from the fact that an effective theory is merely constrained by Ward identities (WI), which fix its *local* but not its global aspects. In particular the usual solution of the anomalous U(1) WI only guarantees $\mathcal{L}(\hat{U}, \hat{\chi}, \theta) = \mathcal{L}(\hat{U}, \hat{\chi}\mathrm{e}^{i\theta/n})$ for $\theta \sim 0$. However, under the $\mathrm{Z_{chiral}}(n)$ transformation one has

$$\mathcal{L}(\hat{U}\mathrm{e}^{-\frac{2\pi ik}{n}}, \hat{\chi}\mathrm{e}^{i\theta/n}) = \mathcal{L}(\hat{U}, \hat{\chi}\mathrm{e}^{i\frac{\theta+2\pi k}{n}})\,, \tag{9}$$

i.e. the above prescription (8) *restores the* 2π*-periodicity* in the vacuum angle [8]. The $\mathrm{Z_{chiral}}(n)$-symmetry of the vacuum expressed in terms of a local Lagrangian amounts to the constraint

$$\mathcal{L}(\hat{U}\mathrm{e}^{-\frac{2\pi ik}{n}},\hat{\chi},\theta)=\mathcal{L}(\hat{U},\hat{\chi},\theta)=\mathcal{L}(\hat{U},\hat{\chi}\mathrm{e}^{i\frac{\theta+2\pi k}{n}})\,, \tag{10}$$

which is not necessarily true in general and it will be taken as a *definition* of the $\mathrm{Z}_{\mathrm{chiral}}(n)$-symmetric phase. The effective Lagrangian exhibiting $\mathrm{Z}_{\mathrm{chiral}}(n)$-symmetry consists of two parts, $\mathcal{L}=\mathcal{L}_1+\mathcal{L}_2$. $\mathcal{L}_1$ has the whole continuous $\mathrm{U}_A(1)$ symmetry, $\hat{\chi}\to\mathrm{e}^{i\alpha}\hat{\chi}$, i.e. it is independent of the θ angle,

$$\mathcal{L}_1=\frac{F^2}{4}\Big\{\langle D_\mu\hat{U}D^\mu\hat{U}^\dagger\rangle+Z\langle\hat{U}^\dagger\hat{\chi}\rangle\langle\hat{\chi}^\dagger\hat{U}\rangle+\ldots\Big\}\,, \tag{11}$$

where dots stand for pure source and higher orders terms. $\mathcal{L}_2$ consists of terms which break $\mathrm{U}_A(1)$ down to $\mathrm{Z}_{\mathrm{chiral}}(n)$. The lowest order term with this property reads

$$\begin{aligned}\mathcal{L}_2=&\sum_k^{n-1}\sum_{\{j_1..j_k\}}A^{(k)}_{\{j_1..j_k\}}\langle\big(\hat{U}^\dagger\hat{\chi}\big)^{j_1}\rangle\ldots\langle\big(\hat{U}^\dagger\hat{\chi}\big)^{j_k}\rangle+\mathrm{h.c}\,,\\ \text{where}\quad & j_1,\ldots,j_k=1,\ldots,n,\quad j_1+\ldots+j_k=n\,.\end{aligned} \tag{12}$$

Despite the fact that the Lagrangians (11) and (12) are of different orders in $\hat{\chi}$ they can coexist, since they describe two independent sectors of the effective theory: Eq. (12) is *holomorphic* and is not renormalized by loops arising from the U(1)-invariant sector as represented by Eq. (11). Eq. (12) becomes more transparent using Eqs. (3)-(4) and expanding in powers of the light quark masses χ. Denoting by W the 3×3 matrix and by ζ the phase factor,

$$W=U^\dagger\chi\mathrm{e}^{-i\frac{\varphi}{3}},\quad \zeta=\mathrm{e}^{i\frac{\theta+\varphi}{N}}\,, \tag{13}$$

Eq. (12) can be rewritten as

$$\begin{aligned}\mathcal{L}_2=&\,a_n(M\zeta)^n+b_n\langle W\rangle(M\zeta)^{n-1}+c_n\langle W^2\rangle(M\zeta)^{n-2}\\ &+d_n\langle W\rangle^2(M\zeta)^{n-2}+\mathrm{h.c.}+\mathcal{O}(W^3)\,.\end{aligned} \tag{14}$$

Similarly, the reduction SU(n)$\to$SU(3)$\times$U(1) of the component $\mathcal{L}_1$ can be expressed in terms of the variables (13) as

$$\begin{aligned}\mathcal{L}_1=&\frac{F^2}{4}\Big\{\langle D_\mu U^\dagger D^\mu U\rangle+\frac{n}{3N}\partial_\mu\varphi\partial^\mu\varphi\\ &+MNZ\langle W\zeta^\dagger+W^\dagger\zeta\rangle+Z\langle W^\dagger\rangle\langle W\rangle\Big\}+\ldots\,.\end{aligned} \tag{15}$$

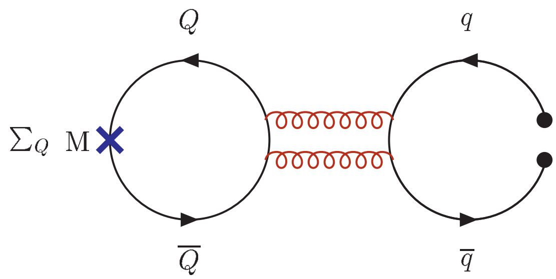

FIG. 1. Representation of the contribution to the induced condensate Eq. (17). The cross refers to $\overline{Q}Q$ insertion.

In the tree approximation the η'-mass merely arises from the holomorphic part $\mathcal{L}_2$, ($m_u = m_d = m_s = 0$),

$$M_{\eta'}^2 = \frac{12N}{n}\frac{1}{F^2}a_n M^n\,. \tag{16}$$

In contrast to the $\mathrm{Z}_{\mathrm{chiral}}(n)$-asymmetric phase, [cf. Eq. (1)], which contains a genuine condensate term B_0, such a term is absent in the $\mathrm{Z}_{\mathrm{chiral}}(n)$-symmetric phase. However, in the reduction (3)-(4), an *induced condensate* appears through the OZI violation terms [see Fig. (1)] (the $\mathrm{Z}_{\mathrm{chiral}}(n)$-symmetry is explicitly broken by the Q-mass term),

$$F^2 B_{\mathrm{induced}} = \frac{1}{2}F^2 MNZ + 2b_n M^{n-1}\,, \tag{17}$$

where the first term on the r.h.s. arises from $\mathcal{L}_1$, whereas the second term comes from the holomorphic Lagrangian. The quadratic terms in W of Eq. (14) contribute to the subleading low-energy constants L_6, L_7, L_8 [9], denoted by hat

$$\begin{aligned} B^2_{\mathrm{induced}}(\hat{L}_6 - \hat{L}_7) &= \tfrac{1}{32}F^2 Z\,,\\ B^2_{\mathrm{induced}}\hat{L}_8 &= \tfrac{1}{4}c_n M^{n-2}\,,\\ B^2_{\mathrm{induced}}(\hat{L}_6 + \hat{L}_7) &= \tfrac{1}{4}d_n M^{n-2}\,. \end{aligned} \tag{18}$$

4. We now turn to the leading behavior of all these induced low-energy constants in the combined large-N limit. We consider (connected) correlators of quark bilinears $\bar{\Psi}\Gamma\Psi$.

Usual large-N_c counting rules are maintained. In addition, every quark loop gives rise to a *flavor trace* involving all flavor matrices contained in that loop. Consequently, each internal ("sea") quark loop will be *enhanced* by a factor $N_f = n$ and suppressed (as usual) by a factor $1/N_c$. (However, if a quark loop is *valence*, i.e. attached to an external flavor source, it will not lead to a flavor-enhancement factor.) In particular, the constants F^2 and Z in Eq. (11) behave as $F^2 \sim N$ and $Z \sim 1/N$. The contribution to the fermionic determinant can be formally written (in a large Euclidean box) like

$$\Delta = \exp \sum_k \log \left(1 + \frac{\lambda_k^2 - \omega_k^2}{\omega_k^2 + M^2}\right)^n , \tag{19}$$

where λ_k are Dirac operator eigenvalues and ω_k the corresponding eigenvalues in the absence of interactions. Hence at large-N_c one expects $\lambda_k^2 - \omega_k^2 \sim \mathcal{O}(g^2) \sim \mathcal{O}(1/N_c)$. This illustrates the mechanism by which the fermionic determinant stays non trivial and finite in the combined large-N limit, merely depending on the ratio N_f/N_c. The large-N counting of the holomorphic part is more subtle: we deal with a large-N_f, large-N_c behavior of a large-N-point function. In the tree approximation, the holomorphic part of the Lagrangian (at $\hat{U} = \mathbf{1}$) is connected with a QCD correlation function in a Euclidean finite volume V

$$\begin{aligned} -\mathcal{L}_2(\mathbf{1}, \hat{\chi}) &= -\sum_k^{n-1} \sum_{\{j_1 \ldots j_k\}} A^{(k)}_{\{j_1 \ldots j_k\}} \langle \hat{\chi}^{j_1} \rangle \ldots \langle \hat{\chi}^{j_k} \rangle \\ &= \frac{1}{n!} \frac{1}{V} \langle \Big[\int dx \bar{\Psi}_L \hat{\chi} \Psi_R(x) \Big]^n \rangle_{\text{con}} . \end{aligned} \tag{20}$$

The average on r.h.s. of Eq. (20) can be evaluated at non zero masses m and M. The other chirality part $\bar{\Psi}_R \Psi_L$ will contribute but *not* to the lowest order $\hat{\chi}^n$ of the holomorphic part of the Lagrangian. Let us introduce the notation

$$\begin{aligned} K &= \textstyle\int dx \sum_{i=1}^N \bar{Q}^i_L(x)\, Q^i_R(x) , \\ k_i &= \textstyle\int dx\, \bar{q}_{iL}(x)\, q_{iR}(x) , \quad i = 1, 2, 3 . \end{aligned} \tag{21}$$

Choosing in Eq. (20) $\hat{\chi} = \text{diag}(m_1, m_2, m_3, M, \ldots, M)$, and combining it with Eq. (14) one gets

$$-\Big\{ a_n M^n + b_n M^{n-1} (m_1 + m_2 + m_3)$$

$$
\begin{aligned}
&+\left[c_n(m_1^2+m_2^2+m_3^2)+d_n(m_1+m_2+m_3)^2\right]M^{n-2} \\
&+\ldots\} = \frac{1}{n!}\frac{1}{V}\langle (MK+m_1k_1+m_2k_2+m_3k_3)^n\rangle_{\rm con}\,.
\end{aligned} \tag{22}
$$

Comparing the coefficients of M^n

$$
a_n M^n = -\frac{1}{n!}M^n\frac{1}{V}\langle K^n\rangle_{\rm con}\,. \tag{23}
$$

The single quark loop contribution to Eq. (23) reads

$$
\begin{aligned}
&\frac{1}{V}\langle K^n\,\rangle = -(n-1)!\frac{1}{V}\langle\langle\int dx_1\ldots dx_n {\rm Tr}\Big\{S^{RL}(x_1,x_2) \\
&S^{RL}(x_2,x_3)\ldots S^{RL}(x_n,x_1)\Big\}\rangle\rangle{\rm Tr}(\mathbf{1}_{N\times N})\,,
\end{aligned} \tag{24}
$$

where $S^{RL}(x,y)$ denotes the chiral part of the fermion propagator

$$
S^{RL}(x,y){=}\left(\frac{1+\gamma_5}{\sqrt{2}}\right)\sum_{\lambda_k\geq 0}\frac{M}{M^2+\lambda_k^2}\varphi_k(x)\varphi_k^\dagger(y)\left(\frac{1+\gamma_5}{\sqrt{2}}\right)
$$

in terms of the orthonormal Fujikawa chiral basis [10] (φ_k is the Dirac eigenvector belonging to the eigenvalue λ_k) and $\langle\langle\ldots\rangle\rangle$ stands for the average over gluon configurations with insertion of fermionic determinant. The factor $(n-1)!$ in Eq. (24) counts the different ways of connecting n points by a single *one quark loop.* In fact a closer examination of the combinatorics of multiloop diagrams' contributions to $\langle K^n\rangle$ in Eq. (24) reveals that none of them is more important that the one with the least number of quark loops. The integrals in Eq. (24) can be performed with the result

$$
a_n M^n \sim \lim_{\substack{V\to\infty \\ n\to\infty}}\frac{1}{V}\langle\langle\sum_{\lambda_k\geq 0}\left(1+\frac{\lambda_k^2}{M^2}\right)^{-n}\rangle\rangle\,. \tag{25}
$$

Even if we do not consider here the chiral limit $M\to 0$, the behavior of Eq. (25) is merely controlled by the average density of small Dirac eigenvalues. Indeed, for any fixed M and (arbitrary small) ϵ the eigenvalues $\lambda_k^2\geq\epsilon$ do not contribute to the large-N limit of Eq. (25). The latter should be of the order of the average number of states $\mathcal{N}_\epsilon$ with $\lambda_k^2\leq\epsilon$. On general grounds one expects $\mathcal{N}_\epsilon\sim VN_c$ [11]. (The density of states should grow proportionally with N_c.) Hence, in the combined large-N limit $a_nM^n\sim\mathcal{O}(N_c)$ and according to Eq. (16)

$$M_{\eta'}^2 \sim \mathrm{const}\,, \tag{26}$$

thus not suppressed anymore. Similar conclusions have been reached also in different contexts [12,13]. The remaining coefficients in Eq. (22) can be found similarly:

$$b_n M^{n-1} \sim -\frac{M^{n-1}}{(n-1)!}\frac{1}{V}\langle K^{n-1}k_1\rangle_{\mathrm{con}}\,, \tag{27}$$

$$(c_n + d_n)M^{n-2} \sim -\frac{M^{n-2}}{(n-2)!}\frac{1}{V}\langle K^{n-2}k_1^2\rangle_{\mathrm{con}}\,, \tag{28}$$

receiving leading contribution from at least two quark loops and consequently suppressed by $1/N_c$ relative to Eq. (25). Finally

$$d_n M^{n-2} \sim -\frac{M^{n-2}}{(n-2)!}\frac{1}{V}\langle K^{n-2}k_1k_2\rangle_{\mathrm{con}}\,, \tag{29}$$

which involves at least three quark loops. This leads to the final estimate

$$b_n M^{n-1} \sim c_n M^{n-2} \sim \mathcal{O}(1)\,, \quad d_n M^{n-2} \sim \mathcal{O}(1/N_c)\,. \tag{30}$$

As a consequence, the holomorphic contribution to the induced condensate, Eq. (17), is suppressed relative to the non-holomorphic one. The latter is given by the OZI rule violating constant Z which is suppressed by $1/N_c$ but this suppression is compensated by a flavor enhancement factor $N = n - 3$. As a result

$$F^2 B_{\mathrm{induced}} \sim \mathcal{O}(N) + \mathcal{O}(1)\,, \tag{31}$$

where the first term is the non-holomorphic and the second one the holomorphic contribution. For the tree contribution quoted in Eq. (18) the large-N counting reads

$$\begin{aligned} &B_{\mathrm{induced}}^2\,(\hat{L}_6 - \hat{L}_7) \sim B_{\mathrm{induced}}^2\,\hat{L}_8 \sim \mathcal{O}(1)\,, \\ &B_{\mathrm{induced}}^2\,(\hat{L}_6 + \hat{L}_7) \sim \mathcal{O}(1/N)\,. \end{aligned} \tag{32}$$

5. More comments on the large-N behavior of the η'-mass are in order. The usual argument for finding the behavior of the η'-mass [2] derives from the necessity to cancel

the θ-dependence of the pure gluodynamics when massless quarks are added. This is only possible when the $1/N_c$ suppression of the internal quark loops is compensated by the η'-pole contribution $M_{\eta'}^{-2}$. This leads to the Veneziano-Witten's formula and the vanishing of $M_{\eta'}^2$ as $1/N_c$. On the other hand, in the combined large-N limit internal quark loops are not suppressed, and it is not possible to isolate pure glue contributions by large-N arguments. As a consequence the η'-mass does not vanish anymore and its relation to the topological susceptibility is lost. We may as well consider the η'-field as heavy and integrate it out. At tree level this amounts to the shift in the constant L_7

$$L_7 = \hat{L}_7 - \frac{F^2}{48 M_{\eta'}^2}, \quad L_i = \hat{L}_i, \quad (i \neq 7)\,, \tag{33}$$

as it is seen by evaluating the singlet minus octet pseudoscalar two-point function [14].

6. Let us summarize the results of this work. We have asked whether the combined large-N limit (as defined in this paper) helps understanding the peculiar properties of QCD in the vacuum and η'-channels. (*i*) In the scalar channel, this limit suggests a flavor enhancement of the OZI rule violation, leading in particular to the emergence of an *induced quark condensate* [Eq. (17)] on top of the genuine condensate B_0 [c.f. Eq. (1)]. In the large N_f/N_c phase in which the genuine condensate is forbidden due to the $\mathrm{Z}_{\mathrm{chiral}}(n)$-symmetry, the induced condensate plays the rôle of B_0 in describing chiral symmetry breaking. An induced condensate would manifest itself by an important flavor dependence [as in Eq. (17)] and it could be, in principle, disentangled from B_0 in this way [3]. (*ii*) A distinctive feature of the $\mathrm{Z}_{\mathrm{chiral}}(n)$-symmetric phase is the non-vanishing η'-mass in the combined large-N limit. (Since this phase is expected for large N_f/N_c, the usual large-N_c, fixed-N_f arguments do not apply.) In particular, the relation of the $M_{\eta'}$ to the axial anomaly and to the topological susceptibility of the pure YM theory are now modified. (*iii*) The place of the scale M in building η'-mass, induced condensate and the low-energy constants L_6, L_7 and L_8 is important and not entirely understood. Special attention should be payed to the rôle of M in the low-energy expansion and in the renormalization of the whole effective theory. We have

calculated the one-loop contribution to all the above mentioned quantities. Qualitatively, they do not modify any of our conclusions.

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